The extended two-type parameter estimator in linear regression model

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Amir Zeinal[1*]

[1] *Department of statistics, Faculty of Mathematical sciences, University of Guilan, Rasht, Iran; E-mail (amirzeinal@guilan.ac.ir)*

Abstract— *In this paper, a new two-type parameter estimator is introduced. This estimator is an extension of the two-parameter estimator presented by Özkale and Kaçiranlar [10], which includes the ordinary least squares, the generalized ridge and the generalized Liu estimators, as special cases. Here the performance of this new estimator over the ordinary least squares and two-parameter estimators is , theoretically, evaluated in terms of quadratic bias (QB) and mean squared error matrix (MSEM) criteria, and the optimal biasing parameters are obtained to minimize the scalar mean squared error (MSE). Then a numerical example is given and a simulation study is done to illustrate the theoretical results of the paper.*

Keywords— Generalized Liu estimator, Generalized ridge estimator, Lagrange method, Mean squared error matrix, Two-parameter estimator.

1. INTRODUCTION

Let us consider the linear regression model

$$Y = X\beta + \varepsilon \qquad (1)$$

, where $Y = (y_1, \ldots, y_n)'$ is random vector of response value,

$X = (X_1, X_2, \ldots, X_n)'$ is an $n \times p$ regressors matrix of full column rank with

$X_i = (X_{i1}, \ldots, X_{ip})'$ for $i = 1, 2, \ldots, n$, β is a $p \times 1$ vector of unknown regression coefficients and ε is an $n \times 1$ vector of error terms with expectation $E(\varepsilon) = 0$ and covariance matrix $\mathrm{Cov}(\varepsilon) = \sigma^2 I_n$.

According to Gauss-Markov theorem, the ordinary least squares (OLS) estimator is obtained as follows:

$$\hat{\beta}_{OLS} = (X'X)^{-1} X'Y.$$

Multicollinearity, linear or near linear dependency among the regressors, in the linear regression model is an important problem faced in applications. If multicollinearity is present, the small relative changes in the matrix $X'X$ will produce large relative changes in the matrix $(X'X)^{-1}$. Thus the OLS estimator results in a large variance and it will not be a precise estimator.

In order to deal with the multicollinearity, Hoerl and Kennard [5] proposed the ordinary ridge (OR) estimator,

$$\hat{\beta}(k) = (X'X + kI)^{-1} X'Y, \quad k > 0.$$

And Liu [7] proposed the Liu estimator which combines the Stein [13] estimator with OR estimator,

$$\hat{\beta}(d) = (X'X + I)^{-1} (X'Y + d\hat{\beta}_{OLS}), \quad 0 < d < 1.$$

As well as the above-mentioned estimators, some other biased estimators were introduced in the literature such as the r-d class estimator [6], the Liu-type estimator [8], the two-parameter estimator [10], the principal component k-d class estimator [2], the r-k class estimator [11, 12], the ridge 2 estimator [14].

In Section 2 of this paper, the two-parameter (TP) estimator presented by Özkale and Kaçiranlar [10] is extended. In Section 3, the performance of the proposed estimator with respect to quadratic bias (QB) and mean squared error matrix (MSEM) criteria is discussed, and in Section 4, a method presented to choose the biasing parameters. To compare this estimator with TP and OLS estimators, a numerical example is presented in Section 5, and a simulation study is done in Section 6.

2. THE PROPOSED ESTIMATOR

In this Section, the extended two-type parameter (ETTP) estimator is introduced. The model (1) can be rewritten in canonical form,

$$Y = Z\alpha + \varepsilon \tag{2}$$

, where $Z = XQ$, $\alpha = Q'\beta$, and Q is the orthogonal matrix whose columns constitute the eigenvectors of $X'X$. Also,

$$Z'Z = Q'X'XQ = \Lambda = diag(\lambda_1, \ldots, \lambda_p)$$

, where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p$ are eigenvalues of $X'X$. The different estimators, from model (2), are obtained, such as

$\hat{\alpha}_{OLS} = \Lambda^{-1}Z'Y$, which is the ordinary least squares estimator.

$\hat{\alpha}(k) = (\Lambda + kI)^{-1}Z'Y,\ k > 0$, which is the ordinary ridge estimator.

$\hat{\alpha}(d) = (\Lambda + I)(Z'Y + d\hat{\alpha}_{OLS}) = (\Lambda + I)^{-1}(\Lambda + dI)\hat{\alpha}_{OLS},\ 0 < d < 1,$ which is the Liu estimator.

The two-parameter estimator introduced by Özkale and Kaçiranlar is defined by

$$\hat{\alpha}(k,d) = (\Lambda + kI)^{-1}(Z'Y + kd\hat{\alpha}_{OLS}) = (\Lambda + kI)^{-1}(\Lambda + kdI)\hat{\alpha}_{OLS},\ k > 0,\ 0 < d < 1. \tag{3}$$

This estimator is derived by minimizing $(Y - Z\alpha)'(Y - Z\alpha)$ subject to

$$(\alpha - d\hat{\alpha}_{OLS})'(\alpha - d\hat{\alpha}_{OLS}) = c,$$

that is by minimizing the following function

$$(Y - Z\alpha)'(Y - Z\alpha) + k\left[(\alpha - d\hat{\alpha}_{OLS})'(\alpha - d\hat{\alpha}_{OLS}) - c\right]$$

where c is a constant and k is a Lagrangian multiplier.

Here, instead of minimizing the above-mentioned function, the following function is minimized.

$$(Y - Z\alpha)'(Y - Z\alpha) + \sum_{i=1}^{p} k_i\left[(\alpha_i - d_i\hat{\alpha}_{i,OLS})'(\alpha_i - d_i\hat{\alpha}_{i,OLS})' - c_i\right]$$

(4)

By introducing $D = diag(d_1,\ldots,d_p)$, $K = diag(k_1,\ldots,k_p)$ and $C^{\frac{1}{2}} = \left[\sqrt{c_1},\ldots,\sqrt{c_p}\right]'$, function (4) equals

$$(Y - Z\alpha)'(Y - Z\alpha) + (D\hat{\alpha}_{OLS} - \alpha)'K(D\hat{\alpha}_{OLS} - \alpha) - \left(C^{\frac{1}{2}}\right)' K C^{\frac{1}{2}}$$

(5)

Differentiating the function in (5) with respect to α leads to

$$(Z'Z + K)\alpha = Z'Y + K\hat{\alpha}_{OLS}.$$

Consequently,

$$\hat{\alpha}(K,D) = (\Lambda + K)^{-1}(Z'Y + KD\hat{\alpha}_{OLS}) = (\Lambda + K)^{-1}(\Lambda + KD)\hat{\alpha}_{OLS}$$

(6)

, where $k_i > 0$ and $0 < d_i < 1$, $i = 1,2,\ldots,p$.

This estimator is defined as extended two-type parameter (ETTP) estimator.

Different estimators are derived from $\hat{\alpha}(K,D)$ as follows:

(I) $\lim_{D \to I} \hat{\alpha}(K,D) = \hat{\alpha}_{OLS}$.

(II) $\lim_{K \to 0} \hat{\alpha}(K,D) = \hat{\alpha}_{OLS}$.

(III) $\lim_{D \to 0} \hat{\alpha}(K,D) = (\Lambda + K)^{-1}\Lambda\hat{\alpha}_{OLS} = (\Lambda + K)^{-1}Z'Y$, which is the generalized ridge estimator.

(IV) $\lim_{D \to 0} \hat{\beta}(kI, D) = \left(\Lambda + kI\right)^{-1} Z'Y$.

(V) $\hat{\beta}(kI, dI) = \hat{\beta}(k, d)$.

(VI) $\hat{\beta}(I, D) = \left(\Lambda + I\right)^{-1}\left(\Lambda + D\right)\hat{\beta}_{OLS}$, which is the generalized Liu estimator.

(VII) $\hat{\beta}(I, dI) = \left(\Lambda + I\right)^{-1}\left(\Lambda + dI\right)\hat{\beta}_{OLS}$.

3. THE PERFORMANCE OF THE NEW ESTIMATOR

In this section, the performance of ETTP estimator was compared with TP estimator by QB criterion, and also was compared with OLS and TP estimators, by MSEM criterion, theoretically.

a. QB CRITERION

$QB(\hat{\beta}) = \text{Bias}(\hat{\beta})'\text{Bias}(\hat{\beta})$, where $\text{Bias}(\hat{\beta}) = E(\hat{\beta}) - \alpha$.

The following equations were resulted from equations (3) and (6).

$$\text{Bias}(\hat{\beta}(k,d)) = \left[\left(\Lambda + kI\right)\left(\Lambda + kdI\right) - I\right]\alpha \tag{7}$$

$$\text{Bias}(\hat{\beta}(K,D)) = \left[\left(\Lambda + K\right)^{-1}\left(\Lambda + KD\right) - I\right]\alpha \tag{8}$$

Consequently,

$$QB(\hat{\beta}(K,D)) - QB(\hat{\beta}(k,d)) = \sum_{i=1}^{p} \frac{\left[k_i(d_i - 1)(\lambda_i + k)\right]^2 - \left[k(d - 1)(\lambda_i + k_i)\right]^2}{(\lambda_i + k_i)^2(\lambda_i + k)^2}\alpha_i^2$$

Thus, the following theorem is resulted:

Theorem 3.1.1: If

$$\frac{\lambda_i + k_i}{k_i(d_i - 1)} < \frac{\lambda_i + k}{k(d - 1)}, \quad i = 1, 2, \ldots, p,$$

then $QB(\hat{\alpha}(K,D)) < QB(\hat{\alpha}(k,d))$.

b. MSEM CRITERION

The mean squared error matrix of $\hat{\alpha}$ is defined as follows:

$$\mathrm{MSEM}(\hat{\alpha}) = \mathrm{Cov}(\hat{\alpha}) + \mathrm{Bias}(\hat{\alpha})\mathrm{Bias}(\hat{\alpha})'.$$

Lemma 3.2.1. (Farebrother [3]). Let M be a positive definite matrix, namely $M > 0$, and let d be some vector, then $M - dd' \geq 0$ if and only if $d'M^{-1}d \leq 1$.

Lemma 3.2.2. (Trenkler and Toutenburg [15]). Let $\hat{\alpha}(j) = A_j Y$, $j = 1, 2$ be two competing estimators of α. Suppose that $D = \mathrm{Cov}(\hat{\alpha}_1) - \mathrm{Cov}(\hat{\alpha}_2) > 0$, where $\mathrm{Cov}(\hat{\beta}_j)$, $j = 1, 2$ denotes the covariance matrix of $\hat{\alpha}_j$, $j = 1, 2$. Then $\Delta(\hat{\alpha}_1, \hat{\alpha}_2) = \mathrm{MSEM}(\hat{\alpha}_1) - \mathrm{MSEM}(\hat{\alpha}_2) \geq 0$ if and only if $d_2'(D + d_1 d_1')^{-1} d_2 \leq 1$, where $\mathrm{MSEM}(\hat{\alpha}_j)$ and d_j denote the mean squared error matrix and bias vector of $\hat{\alpha}_j$, respectively.

It is well-known that

$$\mathrm{Bias}(\hat{\alpha}_{OLS}) = 0$$

(9)

$$\mathrm{Cov}(\hat{\alpha}_{OLS}) = \sigma^2 \Lambda^{-1}$$

(10)

Using equations (6) and (10), the following equation is obtained:

$$\mathrm{Cov}(\hat{\beta}(K,D)) = \sigma^2(\Lambda + K)^{-1}(\Lambda + KD)\Lambda^{-1}(\Lambda + KD)(\Lambda + K)^{-1}$$

By considering

$$\tilde{A} = (\Lambda + K)^{-1}(\Lambda + KD) \tag{11}$$

, it is concluded that

$$\mathrm{Cov}(\hat{\beta}(K,D)) = \sigma^2 \tilde{A}\Lambda^{-1}\tilde{A}' \tag{12}$$

On the other hand, from equations (10) and (12), it is concluded that

$$D_1 = \mathrm{Cov}(\hat{\beta}_{OLS}) - \mathrm{Cov}(\hat{\beta}(K,D)) = \sigma^2\left(\Lambda^{-1} - \tilde{A}\Lambda^{-1}\tilde{A}'\right). \tag{13}$$

And also

$$D_1 = diag\left[\frac{(\lambda_i + k_i)^2 - (\lambda_i + k_i d_i)^2}{\lambda_i(\lambda_i + k_i)}\right]$$

Consequently, if $k_i > 0$ and $0 < d_i < 1$, $i = 1,\ldots,p$, then $D_1 > 0$.

Now, from equations (8), (9), (11) and (13) and Lemma 3.2.2, the following theorem is resulted:

Theorem 3.2.1. If $k_i > 0$ and $0 < d_i < 1$, $i = 1,\ldots,p$, then

$$\mathrm{MSEM}(\hat{\beta}(K,D)) \le \mathrm{MSEM}(\hat{\beta}_{OLS})$$

if and only if

$$\alpha'[\tilde{A}' - I][\sigma^2(\Lambda^{-1} - \tilde{A}\Lambda^{-1}\tilde{A}')]^{-1}[\tilde{A} - I]\alpha \le 1.$$

Using equations (3) and (10), the following equation is obtained:

$$\mathrm{Cov}(\hat{\alpha}(k,d)) = \sigma^2(\Lambda + kI)^{-1}(\Lambda + kdI)\Lambda^{-1}(\Lambda + kdI)(\Lambda + kI)^{-1}$$

By considering

$$\bar{A} = (\Lambda + kI)^{-1}(\Lambda + kdI)$$

(14)

, it is concluded that

$$\mathrm{Cov}(\hat{\alpha}(k,d)) = \sigma^2\bar{A}\Lambda^{-1}\bar{A}'$$

(15)

On the other hand, from equations (12) and (15), it is concluded that

$$D_2 = \mathrm{Cov}(\hat{\alpha}(k,d)) - \mathrm{Cov}(\hat{\alpha}(K,D)) = \sigma^2\left(\bar{A}\Lambda^{-1}\bar{A}' - \tilde{A}\Lambda^{-1}\tilde{A}'\right)$$

(16)

And also

$$D_2 = diag\left\{\frac{(\lambda_i + kd)^2}{\lambda_i(\lambda_i + k)^2} - \frac{(\lambda_i + k_i d_i)^2}{\lambda_i(\lambda_i + k_i)^2}\right\}$$

Consequently, if

$$N_i = (\lambda_i + kd)^2(\lambda_i + k_i)^2 - (\lambda_i + k_i d_i)^2(\lambda_i + k)^2 > 0, \quad i = 1,\ldots,p,$$

then $D_2 > 0$.

Now, noticing that $N_i > 0$ if and only if

$$l_i\left[k_i(1-d_i) - k(1-d)\right] > 0,$$

then, if $k_i(1-d_i) > k(1-d)$, $i = 1,\ldots,p$, $D_2 > 0$ is resulted.

Now, from equations (7), (8), (11), (14) and (16) and Lemma 3.2.2, the following theorem is resulted.

Theorem 3.2.2. If $k(1-d) < \min\{k_i(1-d_i), i = 1,\ldots,p\}$, then

$$\mathrm{MSEM}(\hat{\alpha}(K,D)) \le \mathrm{MSEM}(\hat{\alpha}(k,d)),$$

if and only if

$$\alpha'\left[\tilde{A}' - I\right]\left[\sigma^2\left(\bar{A}\Lambda^{-1}\bar{A}' - \tilde{A}\Lambda^{-1}\tilde{A}'\right) + \left[\bar{A} - I\right]\alpha\alpha'\left[\bar{A}' - I\right]\right]^{-1}\left[\tilde{A} - I\right]\alpha \le 1.$$

4. SELECTION OF THE PARAMETERS k_i AND d_i, $i = 1,\ldots,p$

Another criterion measure of goodness of an estimator is

$$\mathrm{MSE}(\hat{\alpha}) = E\left[(\hat{\alpha} - \alpha)'(\hat{\alpha} - \alpha)\right] = \mathrm{tr}(\mathrm{MSEM}(\hat{\alpha})) = \mathrm{tr}\left[\mathrm{Cov}(\hat{\alpha})\right] + \mathrm{Bias}(\hat{\alpha})'\mathrm{Bias}(\hat{\alpha})$$

Thus, the optimal values for k_i and d_i, $i = 1,\ldots,p$, can be derived by minimizing the following function.

$$
\begin{aligned}
f(K,D) &= \mathrm{MSE}(\hat{\alpha}(K,D)) \\
&= \mathrm{tr}\left[\mathrm{Cov}(\hat{\alpha}(K,D))\right] + \mathrm{Bias}(\hat{\alpha}(K,D))'\mathrm{Bias}(\hat{\alpha}(K,D)) \\
&= \sigma^2 \sum_{i=1}^{p} \frac{(\lambda_i + k_i d_i)^2}{(\lambda_i + k_i)^2 \lambda_i} + \sum_{i=1}^{p} \frac{\left[k_i(d_i - 1)\right]^2 \alpha_i^2}{(\lambda_i + k_i)^2} \\
&= \sum_{i=1}^{p} \frac{\sigma^2(\lambda_i + k_i d_i)^2 + k_i^2 (d_i - 1)^2 \alpha_i^2 \lambda_i}{\lambda_i(\lambda_i + k_i)^2}
\end{aligned}
$$

The values of d_i, $i = 1,\ldots,p$, which minimizes $f(K,D)$ for fixed k_i, $i = 1,\ldots,p$, values can be obtained by differentiating $f(K,D)$ with respect to d_i, $i = 1,\ldots,p$.

$$\frac{\partial f(K,D)}{\partial d_i} = \frac{2\sigma^2 k_i(\lambda_i + k_i d_i) + 2k_i^2(d_i - 1)\alpha_i^2 \lambda_i}{\lambda_i(\lambda_i + k_i)^2}, \quad i = 1,\ldots,p,$$

and equating them to zero. After the unknown parameters σ^2 and α_i, $i = 1,\ldots,p$, are replaced with their unbiased estimators, the optimal estimators of d_i, $i = 1,\ldots,p$, for fixed k_i, $i = 1,\ldots,p$, values will be obtained as follows:

$$\hat{d}_{iopt} = \frac{(k_i \hat{\alpha}_i^2 - \hat{\sigma}^2)\lambda_i}{(\lambda_i \hat{\alpha}_i^2 + \hat{\sigma}^2)k_i}, \quad i = 1,\ldots,p$$

(17)

The k_i, $i = 1,\ldots,p$, values which minimize the $f(K,D)$ can be found by differentiating

$$\frac{\partial f(K,D)}{\partial k_i} = \frac{2\sigma^2(\lambda_i + k_i d_i)(d_i - 1) + 2k_i(d_i - 1)^2 \alpha_i^2 \lambda_i}{(\lambda_i + k_i)^3}, \quad i = 1,\ldots,p,$$

and equating them to zero. After the unknown parameters σ^2 and α_i, $i = 1,\ldots,p$, are replaced with their unbiased estimators, the optimal estimators of k_i, $i = 1,\ldots,p$, for fixed d_i, $i = 1,\ldots,p$, values will be obtained as follows:

$$\hat{k}_{iopt} = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2 - d_i\left(\frac{\hat{\sigma}^2}{\lambda_i} + \hat{\alpha}_i^2\right)}, \quad i = 1, \ldots, p$$

(18)

Theorem 4.1. If

$$\hat{d}_i < \frac{\hat{\alpha}_i^2}{\frac{\hat{\sigma}^2}{\lambda_i} + \hat{\alpha}_i^2}, \quad i = 1, \ldots, p,$$

(19)

then $\hat{k}_{iopt} > 0$, $i = 1, \ldots, p$.

Proof: From (18), it was concluded.

5. NUMERICAL EXAMPLE

In order to illustrate the performance of the new estimator, the dataset originally due to Gruber [4], and later discussed by Akdeniz and Erol [1], is considered. Data found in economics are often multicollinear. Table 5.1 gives Total National Research and Development Expenditures-as a percent of Gross National product by country: 1972-1986. It represents the relationship between the dependent variable Y , the percentage spent by the United States, and the four other independent variables X_1, X_2, X_3 and X_4. The variable X_1 represent the percentage spent by France, X_2, the percentage spent by West Germany, X_3 the percentage spent by Japan, and X_4 the percentage spent by the former Soviet Union.

Table 5.1 Year

	Y	X_1	X_2	X_3	X_4
1972	2.3	1.9	2.2	1.9	3.7
1975	2.2	1.8	2.2	2.0	3.8
1979	2.2	1.8	2.4	2.1	3.6
1980	2.3	1.8	2.4	2.2	3.8
1981	2.4	2.0	2.5	2.3	3.8
1982	2.5	2.1	2.6	2.4	3.7
1983	2.6	2.1	2.6	2.6	3.8
1984	2.6	2.2	2.6	2.6	4.0
1985	2.7	2.3	2.8	2.8	3.7
1986	2.7	2.3	2.7	2.8	3.8

By considering $X = \left[X_1, X_2, X_3, X_4\right]$, the eigenvalues of $X'X$ are obtained as follows

$$\lambda_1 = 302.9626,\ \lambda_2 = 0.7283,\ \lambda_3 = 0.0446,\ \lambda_4 = 0.0345.$$

Consequently, the condition number is obtained 8776.381, which suggests the presence of severe collinearity.

In Table 5.2, estimated QB and MSE of OLS, TP and ETTP estimators are presented. To obtain these values, first the theoretical values of the QB and MSE of the estimators were used and then σ^2 and $\alpha_i,\ i = 1,\ldots,p$ were replaced with their unbiased estimators and at last the estimated optimal of their other parameters were used.

Table 5.2

	EMSE	EQB
OLS	0.0539	0
TP	0.0483	0.1289
ETTP	0.0359	0.0094

6. THE MONTE CARLO SIMULATION

To further illustrate the behavior of new estimator, a Monte Carlo simulation study is performed under different levels of multicollinearity.

Following McDonald and Galarneau [9], the explanatory variables are generated by

$$x_{ij} = (1-\rho^2)^{\frac{1}{2}} z_{ij} + \rho z_{ip+1}, \ i = 1,\ldots,p, \ j = 1,\ldots,p$$

where z_{ij}'s are independent standard normal pseudo-random numbers and ρ is specified so that the theoretical correlation between any two explanatory variables is given by ρ^2. Four different values of ρ specified as $0.7, 0.8, 0.9$ and 0.95 are considered.

Observations on the dependent variable are determined by

$$y_i = \beta_1 x_{i1} + \cdots + \beta_p x_{ip} + \varepsilon_i, \ i = 1,\ldots,n,$$

where ε_i's are independent normal pseudo-random numbers with the mean 0 and variance σ^2. The values of σ^2 are considered as $0.2, 2$ and 5.5. Also $\beta_1 = 0.2, \beta_2 = 0.3, \beta_3 = 0.4$ and $\beta_4 = 0.5$ are considered. For each choice of ρ and σ^2, simulation is replicated 10000 times. The estimated mean squared error (EMSE) is calculated for $\hat{\alpha}_{OLS}, \hat{\alpha}(k,d)$ and $\hat{\alpha}(K,D)$ as follows:

$$\text{EMSE}(\hat{\alpha}) = \frac{1}{10000}\sum_{r=1}^{10000} (\hat{\alpha}(r) - \alpha)'(\hat{\alpha}(r) - \alpha).$$

The estimated Bias(EB) is calculated for $\hat{\alpha}(k,d)$ and $\hat{\alpha}(K,D)$ as follows:

$$\mathrm{EB}(\hat{\beta}) = \frac{1}{10000}\sum_{r=1}^{10000}(\hat{\beta}(r) - \beta).$$

Consequently, EQB is calculated as follows:

$$\mathrm{EQB}(\hat{\beta}) = \mathrm{EB}(\hat{\beta})'\mathrm{EB}(\hat{\beta})$$

$\hat{\beta}(r)$, in the above equations, is $\hat{\beta}$ estimator for each replication of the simulation. For each replication, the values k_i and d_i, $i = 1,\ldots,p$, corresponding $\hat{\beta}(K,D)$ are estimated using the method in Section 4. And the values k and d, corresponding $\hat{\beta}(k,d)$ are estimated using the method proposed by Özkale and Kaçiranlar [10]. The EMSE vales and EQB values are presented in Tables 6.1-6.4 and 6.5-6.8, respectively.

Table 6.1- EMSE, $\rho = 0.7$

		σ^2	
	0.2	2	5.5
OLS	0.0133	0.1298	0.3605
TP	0.0133	0.1063	0.2502
ETTP	0.0129	0.0828	0.2035

Table 6.2- EMSE, $\rho = 0.8$

		σ^2	
	0.2	2	5.5
OLS	0.0185	0.1822	0.5054
TP	0.0183	0.1370	0.3220
ETTP	0.0172	0.1081	0.2699

Table 6.3- EMSE, $\rho = 0.9$

		σ^2	
	0.2	2	5.5
OLS	0.3440	0.3376	0.9358
TP	0.3240	0.2178	0.5278
ETTP	0.0281	0.1814	0.4697

Table 6.4- EMSE, $\rho = 0.95$

		σ^2	
	0.2	2	5.5
OLS	0.0650	0.6473	1.8139
TP	0.0568	0.3734	0.9234
ETTP	0.0454	0.3261	0.8883

Table 6.5- EQB, $\rho = 0.7$

		σ^2	
	0.2	2	5.5
TP	1.6850×10^{-4}	0.0073	0.0223
ETTP	1.0663×10^{-4}	6.3563×10^{-4}	0.0017

Table 6.6- EQB, $\rho = 0.8$

		σ^2	
	0.2	2	5.5
TP	3.0530×10^{-4}	0.0109	0.0287
ETTP	1.9256×10^{-4}	7.2132×10^{-4}	0.0011

Table 6.7- EQB, $\rho = 0.9$

		σ^2	
	0.2	2	5.5
TP	8.5201×10^{-4}	0.0215	0.0427
ETTP	3.3033×10^{-4}	8.5389×10^{-4}	9.5165×10^{-4}

Table 6.8- EQB, $\rho = 0.95$

		σ^2	
	0.2	2	5.5
TP	0.0049	0.0353	0.0558
ETTP	3.7517×10^{-4}	4.0973×10^{-4}	0.0011

7. CONCLUSION

In this paper, the two parameter estimator, introduced by Özkale and Kaçiranlar was extended. The performance of the new estimator was compared with OLS and TP estimators, theoretically. And also, by using a numerical example and studying the simulation, it was shown that the performance of the new estimator, in terms of EMSE criterion is better than OLS and TP estimators, and, in terms of EQB criterion is better than TP estimator.

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